Chapter 4: Kidney Exchange

Fuhito Kojima

February 12, 2009

Yale University. http://sites.google.com/site/fuhitokojimaeconomics/.
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Kidney Exchange

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1. 10659 transplants from diseased donors,
2. 6428 transplants from living donors, while
3. 3875 patients died while on the waiting list.
Kidneys cannot be bought and sold

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“it shall be unlawful for any person to knowingly acquire, receive or otherwise transfer any human organ for valuable consideration for use in human transplantation.”
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Given that constraint, donation is the most important source of kidneys.
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2. **Living donors:** Living donors usually come from friends or relatives of a patient (because the monetary transaction is prohibited). Live donation has been increasing recently.

<table>
<thead>
<tr>
<th>Donor Types</th>
<th>2008</th>
<th>1998</th>
<th>1988</th>
</tr>
</thead>
<tbody>
<tr>
<td>All donors</td>
<td>10,920</td>
<td>9,761</td>
<td>5,693</td>
</tr>
<tr>
<td>Deceased donors</td>
<td>5,992</td>
<td>5,339</td>
<td>3,876</td>
</tr>
<tr>
<td>Live donors</td>
<td>4,928</td>
<td>4,422</td>
<td>1,817</td>
</tr>
</tbody>
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**Table:** Number of donors by donor types. Data obtained at [http://www.optn.org/](http://www.optn.org/)
Figure: Live donors by relationship to patients (thanks to Al Roth for providing the graph).
For a successful transplant, the donor kidney needs to be **compatible** with the patient.
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Blood type compatibility: There are four blood types, O, A, B and AB.

- O type patients can receive kidneys from O type donors
- A type patients can receive kidneys from O or A type donors
- B type patients can receive kidneys from O or B type donors
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A problem with transplant from live donors: transplant is carried out if the donor kidney is compatible with the patient. **Otherwise the willing donor goes home and the patient cannot get transplant.**

Is there any way to increase the number and quality of transplant?
A paired exchange: Match two patient-donor pairs (say pair 1 and 2) where
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In such a case, the donor 1 can give her kidney to the patient 2 and the donor 2 can give his kidney to the patient 1 in return.
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Take a look at the web page of Alliance for Paired Donation at http://www.paireddonation.org/anim.htm
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Potential issues include

1. Efficiency (Pareto efficiency; maximizing number of transplantation)
2. Incentives (Strategy-proofness)
3. Fairness
Do patients and doctors behave strategically? Here is one example indicating they do.
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A news report by Reuters (2003-7-29)

*Three Chicago hospitals were accused of fraud by prosecutors on Monday for manipulating diagnoses of transplant patients to get them new livers. Two of the institutions paid fines to settle the charges. “By falsely diagnosing patients and placing them in intensive care to make them appear more sick than they were, these three highly regarded medical centers made patients eligible for liver transplants ahead of others who were waiting for organs in the transplant region,” said Patrick Fitzgerald, the U.S. attorney for the Northern District of Illinois.*
A kidney exchange model is composed of

1. A set of donor-patient (kidney-transplant) pairs \{ (k_1, t_1), ..., (k_n, t_n) \}
2. A preference over \{ k_1, ..., k_n \} \cup \{ w \} for each \( t_i \), where \( w \) is priority in the waitlist (in exchange of donating kidney \( k_i \)).

A matching is a function that specifies which patient obtains which kidney (or waitlist). We assume \( w \) can be matched with any number of patients.

A mechanism is a procedure to select a matching for each problem.
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1. There is no limit on the number of pairs participating in one exchange.

2. Patients have strict preferences over compatible kidneys and the waitlist: Some justification by Opelz (1997). He shows that, in his data, increase in the number of HLA mismatch decreases the likelihood of kidney survival. Other characteristics such as body size and donor age affect kidney survival.
With the assumption of RSU (2004), the kidney exchange problem is mathematically (not necessarily substantively!) very similar to house allocation with existing tenants:
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One difference is that the waitlist $w$ can be matched to multiple patients, but this can be accommodated straightforwardly.
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3. If at any step a cycle forms, assign these kidneys by letting them exchange, and then proceed with the algorithm.
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Remarks: (1) In RSU they consider many variants of the TTC mechanism, which they call TTCC (Top Trading Cycles and Chains) mechanisms. The one which they pick as the winner corresponds to TTC (this point is formally pointed out by Krishna and Wang 2007).
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(2) Sonmez and Unver (2008) give axiomatic characterization of the TTC mechanism, thus adding one more justification of using this mechanism.
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4. Compatible pairs may not participate in an exchange.
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2. $R = (r_{ij})_{i,j \in N}$: the mutual compatibility matrix, that is,

$$r_{ij} = \begin{cases} 
1 & \text{if } i \text{ and } j \text{ are mutually compatible}, \\
0 & \text{otherwise} \end{cases}$$
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A mechanism is strategy-proof if no pair benefits by misreporting who is mutually compatible with them.
Efficiency Property

Proposition (Lemma 1 of RSU (2005))

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This claim does not hold if larger exchanges are possible.
Consider the following priority mechanism (serial dictatorship):

1. Order pairs in some way (ordering could be random or favor waiting time, etc.),
2. If there is any matching in which the top priority pair is matched, then match that pair. Otherwise, skip that pair.
3. Match the second-top priority pair if there is such a matching that also match the first pair (if they were matched in the previous step), then match the pair. Otherwise, skip that pair.
4. Match the $k$th top priority pair if there is such a matching that also match all the pairs that were matched in previous steps, then match the pair. Otherwise, skip that pair.
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The reason for this claim is very intuitive (why?).
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Still, the logistical constraints are likely to matter: two-way (pairwise) exchanges are easier than three-way exchanges, and three-way exchanges are easier than four-way exchanges, and so on.
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How much efficiency gain can we obtain through larger exchanges?
**Example:** A pair is denoted as type x-y if the patient and donor are ABO blood-types x and y, respectively. Consider a population composed of


If only two-way exchanges are possible:


If three-way exchanges are also feasible:

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2. O-type donors can facilitate three transplants rather than two.
Four-way Exchanges Can Add Only A Little

**Example:** Consider a population composed of 

1. **O-A, A-B, B-AB (blood-type incompatible),**
2. **AB-O (positive crossmatch).**

- If only two-way and three-way exchanges are possible: $(O-A, A-B, AB-O)$.
- If four-way exchanges are also feasible: $(AB-O, O-A, A-B, A-AB)$. 

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**Question:** What about 5-way exchanges? What about even larger exchanges?
For theoretical analysis, RSU make a few assumptions.
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1. No patient is tissue-type incompatible with another patient’s donor.

2. Patient-donor pairs of types O-A, O-B, O-AB, A-AB, and B-AB are on the “long side” of the exchange in the sense that at least one pair of each type remains unmatched in each feasible set of exchanges.

3. \( \#(A-B) > \#(B-A) \).

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Theorem (RSU 2007)

Consider a patient population for which Assumptions 1, 2, 3 and 4 hold and let $\mu$ be any maximal matching (when there is no restriction on the size of the exchanges). Then there exists a maximal matching $\nu$ that consists only of two-way, three-way, and four-way exchanges, under which the same set of patients get transplant as in matching $\mu$. 
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The Theorem means that four-way exchanges suffice: all efficient matching can be achieved just using two-way, three-way and four-way exchanges.
A report about a 6-way exchange
(http://www.thebostonchannel.com/health/11320508/detail.html#)
A rare six-way surgical transplant was a success in Boston.
NewsCenter 5’s Heather Unruh reported Wednesday that three people donated their kidneys to three people they did not know. The transplants happened one month ago at Massachusetts General Hospital and Beth Israel Deaconess. The donors and the recipients met Wednesday for the first time.
<table>
<thead>
<tr>
<th>Pop. size</th>
<th>Method</th>
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<tr>
<td></td>
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<td>Two-way</td>
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<tr>
<td>Simulation</td>
<td>8.86</td>
<td>11.272</td>
</tr>
<tr>
<td>(3.4866)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>n = 25</td>
<td>Upperbound 1</td>
<td>12.5</td>
</tr>
<tr>
<td>(3.6847)</td>
<td></td>
<td></td>
</tr>
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<td>(3.8599)</td>
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<tr>
<td>Simulation</td>
<td>21.792</td>
<td>27.266</td>
</tr>
<tr>
<td>(5.0063)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>n = 50</td>
<td>Upperbound 1</td>
<td>27.1</td>
</tr>
<tr>
<td>(5.205)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Upperbound 2</td>
<td>23.932</td>
<td>29.136</td>
</tr>
<tr>
<td>(5.5093)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Simulation</td>
<td>49.708</td>
<td>59.714</td>
</tr>
<tr>
<td>(7.3353)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>n = 100</td>
<td>Upperbound 1</td>
<td>56.816</td>
</tr>
<tr>
<td>(7.2972)</td>
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<td></td>
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<tr>
<td>Upperbound 2</td>
<td>53.496</td>
<td>61.418</td>
</tr>
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<td>(7.6214)</td>
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Notes: Simulation results about average number of patients actually matched and predicted by the formulae to be matched. The standard errors of the population are reported in parentheses. The standard errors of the averages are obtained by dividing population standard errors by square root of the simulation number, 22.36.
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Many mechanisms satisfy consistency, and hence are strategy-proof.
Computational Issues

Is it easy to find a maximal matchings?

Finding a maximal two-way matching is relatively easy. Finding a maximal matching with three-way and up is known to be computationally difficult (NP-complete).

Abraham, Blum, and Sandholm (2007) present an algorithm to find the maximal matchings. According to them, their algorithm is fast enough to use for 10000 pairs or so (remember around 5000 live donations currently).
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When there is no limit on size of the exchange, sometimes it is optimal not to conduct all the currently available exchanges and wait until more patients can be matched.
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Take a look at the web page of Alliance for Paired Donation at http://www.paireddonation.org/anim2.htm

1. In July 2007, the Alliance for Paired Donation started the first of these chains when an altruistic donor in Michigan donated his kidney to a woman in Phoenix, Arizona.

2. As of the end of September this first NEAD chain was at 4 transplants (M. in MI gave to B. in AZ whose husband R. gave to An. in Toledo, whose mom La. gave to Ce. in Columbus whose daughter Li. gave to G. in Columbus simultaneously with Ce.’s transplant, and now G’s sister Av. is the next bridge donor) . . . (3 bridge donors donated so far)

3. The APD started a second NEAD chain on Dec 7, 2007 with a NDD T who gave to D in Columbus whose daughter M gave to S in Orlando, whose daughter E flew to Toledo to give to R from Tennessee which didn’t work, but she bridged instead to MT in Toledo, whose daughter A will be the next bridge donor (3 transplants so far, 1 from a bridge donor)
More issues

How to incorporate compatible pairs?

Weighting different transplants (how good the match is, transportation cost, etc.)

How to organize a transplantation network when there are many transplant centers?

Stochastic mechanisms and fairness (RSU 2005, Yilmaz 2008).
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A lot of unresolved issues.
Reading for the next topic

We will learn school choice. We will see that a lot of techniques are useful: Both two-sided matching and one-sided matching theories are used for designing school choice. The main paper is


I will also talk about the following papers:


