Chapter 5: School Choice

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School Choice: Overview

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Typical goals of school authorities are: (1) efficient placement, (2) fairness of outcomes, (3) easy for participants to understand and use, etc.
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Many studies are currently conducted to evaluate the current school choice mechanisms, and several mechanisms are proposed to improve the outcome.
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The outcome is a matching, which specifies which student attends which school.
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A matching is **stable** if there is

1. **No blocking individual.** \( \mu(s) \) is acceptable to each student \( s \), each \( s \in \mu(c) \) is acceptable to \( c \) for each school \( c \), and \( |\mu(c)| \leq q_c \).
2. **No blocking pair.** There is no pair \( s \) and \( c \) such that \( c \succ_s \mu(s) \) and \( |\mu(c)| < q_c \) and \( s \succ_c \emptyset \), or \( s \succ_c s' \) for some \( s' \in \mu(c) \).
Stability as Fairness Criterion

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So stability may be a reasonable property we want for school choice mechanisms.
The **Boston mechanism:**

1. **Step 0:** Each student submits a preference ranking of the schools.
2. **Step 1:** In Step 1 only the top choices of the students are considered. For each school, consider the students who have listed it as their top choice and assign seats of the school to these students one at a time following their priority order until there are no seats left or there is no student left who has listed it as her top choice.
3. **Step k:** Consider the remaining students. In Step k only the k\textsuperscript{th} choices of these students are considered. For each school still with available seats, consider the students who have listed it as their k\textsuperscript{th} choice and assign the remaining seats to these students one at a time following their priority order until there are no seats left or there is no student left who has listed it as her k\textsuperscript{th} choice.
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2. Moreover, it is easy to manipulate it. Even if a student has a very high priority at a school, unless she lists it as her top choice she loses her priority to students who have top ranked that school.
Worries in Boston mechanism are real

St. Petersburg Times (09/14/2003):
Make a realistic, informed selection on the school you list as your first choice. It's the cleanest shot you will get at a school, but if you aim too high you might miss. Here's why: If the random computer selection rejects your first choice, your chances of getting your second choice school are greatly diminished. That's because you then fall in line behind everyone who wanted your second choice school as their first choice. You can fall even farther back in line as you get bumped down to your third, fourth and fifth choices.

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But the preference revelation game induced by the Boston mechanism is a "coordination game" among large numbers of parents in which there is incomplete information. So it may be unrealistic to expect to reach a Nash equilibrium in practice.
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Given deficiency of the popular Boston mechanism, what mechanism should we use instead?
Student-Proposing DA in School Choice

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Step 1: (a) Each student “applies” to her first choice school.
(b) Each school tentatively holds the applicants with highest priority up to its quota (if s/he is acceptable) and rejects all other students.

Step $t \geq 2$: (a) Each student rejected in Step $(t - 1)$ applies to her next highest choice.
(b) Each school considers both new applicants and the student (if any) held at Step $(t-1)$, tentatively holds the applicants with highest priority up to its quota from the combined set of students, and rejects all other students.

Terminate when no more applications are made. Termination happens in finite time.
Difference of school choice from two-sided matching

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Given these differences, we can see some old results in new lights.
Theorem (Gale and Shapley 1962; RS Theorem 2.12)
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We also learned the student-optimal stable matching is the unanimously worst stable matching for schools, but it is not costly any more (because we do not care about school’s “welfare”).
DA is strategy-proof in school choice problems

Theorem (Dubins and Freedman 1981, Roth 1982; extended by Hatfield and Kojima forthcoming)

The student-proposing DA is (group) strategy-proof. That is, telling the truth is a dominant strategy for every student (and even a joint deviation by a group of students cannot make everyone better off).
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In fact, DA is the only strategy-proof and stable mechanism.
Other good properties of DA

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Balinski and Sonmez (1999) shows that the mechanism used for college admission in Turkey is equivalent to the school-proposing DA, and advocated the change of the mechanism to the student-proposing DA.
Efficiency cost of stability

Let $S = \{i, j, k\}$, $C = \{a, b\}$, and student preferences are

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\succ_i : b, a, \\
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and both schools have one position and priorities are

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so DA is Pareto inefficient.
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If the school districts can tolerate somewhat unfair matchings, how can we design a more efficient mechanism?
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3. There is at least one cycle (why?). Every student in a cycle is assigned a seat at the school she points to and is removed. The counter of each school in a cycle is reduced by one and if it reduces to zero, the school is also removed. Counters of all other schools are unchanged.
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TTC allows students to trade priorities, starting with the students with highest priorities.
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and the school choice TTC inherits good properties from these, as shown by the Theorem.
Example of TTC

The same example as before. $S = \{i, j, k\}$, $C = \{a, b\}$, and student preferences are

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a Pareto efficient matching.

Compare this with the result of DA,

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So far, Boston and New York City have explicitly worked with economists and designed their school choice mechanisms.
Boston School Match (Abdulkadiroglu, Pathak, Roth and Sonmez 2005, 2008)

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Students have priorities at schools set by the school system:
1. Students who already attend the school,
2. Students who live in a walk zone and have their siblings already attending the school,
3. Students whose siblings are already attending the school,
4. Students who live in a walk zone,
5. All other students.
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Priorities are weak, i.e., there are many students in each priority class: This is going to be important (later topic) but for now let’s ignore the issue.
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Chen and Sonmez (2005): experimental evidence on preference manipulation under Boston mechanism.
Advice from the West Zone Parent’s Group meeting, 10/27/03

One school choice strategy is to find a school you like that is undersubscribed and put it as a top choice, OR, find a school that you like that is popular and put it as a first choice and find a school that is less popular for a “safe” second choice.
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Such behavior is clearly a bad choice, and people suffer from not being sophisticated enough to game the system (Abdulkadiroglu et al. advocate the idea that strategy-proofness is a certain fairness criterion).
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DA was implemented in Boston in 2006 and is in use.

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The old NYC system was decentralized:

1. Each student can submit a list of at most 5 schools.
2. Each school obtains the list of students who listed it, and independently make offers.
3. There were waiting lists (run by mail), and 3 rounds of move waiting lists.

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The old NYC system was decentralized:

1. Each student can submit a list of at most 5 schools.
2. Each school obtains the list of students who listed it, and independently make offers.
3. There were waiting lists (run by mail), and 3 rounds of move waiting lists.

Problems with the old system:

1. The system left 30,000 children unassigned to any of their choices and they are administratively assigned.
2. Strategic behavior by schools: school principals were concealing capacities (Sonmez 1997; further studies by Konishi and Unver 2006; Kojima 2008).
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*Before you might have had a situation where a school was going to take 100 new children for 9th grade, they might have declared only 40 seats and then placed the other 60 children outside the process.*
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So, unlike Boston, the market seems to be really two-sided, i.e., we should treat both students and schools as strategic players.
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2. The student-proposing DA is strategy-proof for students: it is a dominant strategy for every student to report true preferences (Dubins and Freedman 1981; Roth 1982; generalized by Hatfield and Milgrom 2005 and Hatfield and Kojima forthcoming GEB).

3. There is no stable mechanism that is strategy-proof for schools (Roth 1982)
4. When the market is large, it is almost strategy-proof for schools to report true preferences (Roth and Peranson 1999; Kojima and Pathak 2008). Recall there are 90,000 students and over 500 public high schools in New York City.
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Abdulkadiroglu et al. and NYC Department of Education changed the mechanism to the student-proposing DA, except for some details:

1. Students can rank only 12 schools.
2. Seats in a few schools, called specialized high schools (such as Stuyvesant and Bronx High School of Science), is assigned in an earlier round, separately from the rest.
3. Some top students are granted to get into a school when they rank the school as their first choices.
4. All unmatched students in the main round will be assigned in the supplementary round, where the random serial dictatorship is used.

These features come from historical constraints and could not be changed. This makes it technically incorrect to use standard results in two-sided matching, but they seem to be small enough a problem (it may be interesting to study if this is true and why or why not.)
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3,000 students did not receive any school they chose, a decrease from 30,000 who did not receive a choice school in the previous year.
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Then, stability is desirable only to the extent that priorities convey societal preferences. On the other hand, stable mechanisms typically results in inefficient matchings (remember TTC versus DA).
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So, what is the efficiency cost of stable mechanisms? We use an axiomatic approach to investigate this question.
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On the other hand, DA at least satisfies a minimal efficiency criterion, non-wastefulness: A student cannot get into a school only if all the seats at that school are allocated to other agents.

Non-wastefulness is a minimal efficiency requirement, and is satisfied by most reasonable mechanisms.
Main Axiom-Competitive Claim Monotonicity

The new axiom: competitive claim monotonicity.
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The new axiom: **competitive claim monotonicity**.

Consider two scenarios:

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Fuhito Kojima  
Chapter 5: School Choice
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1. A student report her preferences on schools $a$, $b$, and receives her second choice $b$. 

   - We say that a mechanism satisfies competitive claim monotonicity if everyone is made weakly better off in the second scenario. That is, competitive claim monotonicity requires that less competition in claiming rights for schools benefits all agents. Another way to understand this axiom: when a student applies (place claims) to schools for which she has no chance, doing so may not hurt herself but hurt others, and cause inefficiency.
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Example: DA satisfies competitive claim monotonicity

Let $S = \{i, j, k\}$, $C = \{a, b\}$, each school has one position and

$\succ_i : b, a,$

$\succ_j : a,$

$\succ_k : a, b,$

$\succ_a : i, j, k,$

$\succ_b : k, i.$

DA results in $\mu = \{(i, a), (j, \emptyset), (k, b)\}.$

If $j$ declares that no school is acceptable (places less competitive claims), then DA results in $\mu' = \{(i, b), (j, \emptyset), (k, a)\},$ Better for everyone!
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Better for everyone!
More generally, DA turns out to satisfy competitive claim monotonicity.
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Intuition: when a student applies to a no chance school, that still may cause an additional “rejection chain” — a chain reaction of application and rejections by a displaced students and schools getting additional applications — so more students are rejected, being forced to apply to less preferred schools.
Characterization Result

The surprising thing is that non-wastefulness and competitive claim monotonicity capture all the content of DA. That is,
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**Theorem**

An allocation rule \( \varphi \) satisfies non-wastefulness and competitive claim monotonicity if and only if there exists a priority structure such that \( \varphi \) is equivalent to a DA with respect to that priority structure.
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**Theorem**

An allocation rule $\varphi$ satisfies non-wastefulness and competitive claim monotonicity if and only if there exists a priority structure such that $\varphi$ is equivalent to a DA with respect to that priority structure.

So, the decision of school districts to use DA is essentially to allow for efficiency cost because of competitive claim monotonicity, and nothing more.
We also consider situations where the priority structure is a primitive, that is, stability with respect to the priority structure is imposed exogenously.
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Let there be an exogenously given priority structure, and \( \phi \) be a stable mechanism. \( \phi \) is DA for that priority structure if and only if it satisfies competitive claim monotonicity.
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**Theorem**

Let there be an exogenously given priority structure, and \( \varphi \) be a stable mechanism. \( \varphi \) is DA for that priority structure if and only if it satisfies competitive claim monotonicity.

So, among stable mechanisms, using DA is the same thing as imposing competitive claim monotonicity.
Based on Ergin (2002).
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Another question to ask is, when is DA costly in school choice? What priority structure ensures efficiency of DA?
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To investigate this issue, it is useful to see (now familiar) example where DA is inefficient.
Example: DA causes inefficiency

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Ergin says the priority structure of the schools is **acyclic** if there is no such cycle (the definition is a little more complicated because he considers many-to-one matching, but basic idea is the same).
Theorem (Ergin 2002)

*DA is Pareto efficient for all possible student preferences if and only if the priority structure of the schools is acyclic.*
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Taken together, it is rare for DA to have no efficiency cost, and most likely there is tension between stability and efficiency.
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Summary

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DA are used in NYC and Boston after economists and policy makers collaborated in design.
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DA are used in NYC and Boston after economists and policy makers collaborated in design.

If stability is not imperative, a different mechanism like TTC may make more sense.
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