Here are some material related to questions I got in and after today’s class. These are good questions, thanks a lot. I encourage you to ask questions!

(1) I showed an example where there is no stable matching in the presence of couples in addition to single students. A question I got is whether there always exists a stable matching if there are only couples and no single students. I said it is still possible for there to exist no stable matching but I didn’t write down an example. Here it is:

There are $C = \{c_1, c_2, c_3\}$ and two couples, $(m_1, w_1)$ and $(m_2, w_2)$.

\[
\succ_{(m_1, w_1)} : (c_1, c_3), (c_2, c_3), \\
\succ_{(m, w)} : (c_1, c_2), \\
\succ_{c_1} : m_2, m_1 \\
\succ_{c_2} : m_2, w_2. \\
\succ_{c_3} : w_1.
\]

It is tedious but easy to show that each matching is unstable, that is, there are coalitions made of a couple of one or two colleges that want to block (deviate from) the proposed matching. If you are interested, you can see somewhat more realistic (and more complicated) example in Roth and Sotomayor’s book, page 142. You can also find some historical account of couples problem in the book too.
So we’re in a very difficult situation: Couples cause problems! Next week, I will discuss possible ways to cope with this problem, which is arguably still unsolved!

(2) Another question I’d like to share with you is “what is the example of an unstable mechanism used in real world?” I was tempted to show such an example in the class but I didn’t have enough time, but it is certainly a great question. One real-life example is the class of “priority mechanisms” used in medical match of Newcastle and Birmingham in the past, among others. Roughly speaking, it tries to match a pair who mutually rank each other as 1st choice (call it 1st-1st choice pair), then 1st-2nd choice pair, 2nd-1st choice pair, etc. If you have Roth and Sotomayor’s book, it is in section 5.5.1.1 of the book.

Another more recent example is the “Boston mechanism” in the context of school choice. This is a mechanism that used to be used in the city of Boston for placing students to schools. It’s not in RS textbook (as it is too new to be there), but you can find a description in Atila Abdulkadiroglu and Tayfun Sonmez (2003) “School Choice: A Mechanism Design Approach” American Economic Review, 93, 729-747 (this paper is very easy to read and it’s fun). I’ll talk about it in several weeks too.

Hope that helps. See you next week!